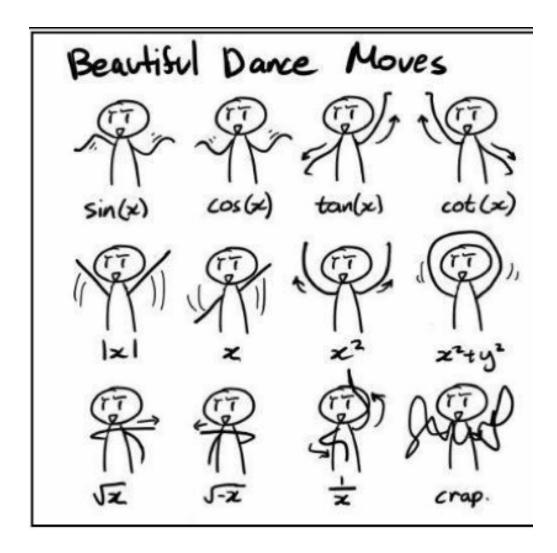
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# **AP Calculus Summer Review Assignment**

- 1. This packet is to be handed in to your Calculus teacher on the first day of the school year.
- 2. All work must be shown in the packet OR on separate paper attached to the packet.



Dear Student and Parent/Guardian,

The math department at Northeast High School wants you to be successful in AP Calculus. This summer packet is designed to help you reach these goals by reviewing necessary skills.

Be sure to follow the key information below when completing this packet:

- •The packet is due when you return to school in August.
- •Every problem must be completed. None left blank.
- •The packet will be collected and graded on the first day of class—no excuses. Work must be shown to receive credit no work, no points. Final answers must be circled.
- •You MUST have the unit circle and the chart memorized. There will be a test on the first day of class.

Use any resources available to you: Internet, Text Books, etc.

We hope that you have an enjoyable summer and return to school ready to be successful in AP Calculus!

Sincerely, Ms. Duszynski Duszynskili@pcsb.org

Helpful Websites www.glencoe.com www.wolframalpha.com www.purplemath.com/modules www.khanacademy.org apcentral.collegeboard.org

#### **Formula Sheet**

Reciprocal Identities: 
$$\csc x = \frac{1}{\sin x}$$
  $\sec x = \frac{1}{\cos x}$   $\cot x = \frac{1}{\tan x}$ 

Quotient Identities: 
$$\tan x = \frac{\sin x}{\cos x}$$
  $\cot x = \frac{\cos x}{\sin x}$ 

Pythagorean Identities: 
$$\sin^2 x + \cos^2 x = 1$$
  $\tan^2 x + 1 = \sec^2 x$   $1 + \cot^2 x = \csc^2 x$ 

Double Angle Identities: 
$$\sin 2x = 2\sin x \cos x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$= 2\cos^2 x - 1$$

Logarithms: The Zero Exponent: 
$$x^0=1$$
, for x not equal to 0.  
 $y = \log_a x$  is equivalent to  $x = a^y$ 

**Multiplying Powers** 

Product property: 
$$\log_b mn = \log_b m + \log_b n$$
 Multiplying Two Powers of the Same Base:  $(\mathbf{x}^a)(\mathbf{x}^b) = \mathbf{x}^{(a+b)}$ 

Quotient property: 
$$\log_b \frac{m}{n} = \log_b m - \log_b n$$
 Multiplying Powers of Different Bases:  $(xy)^a = (x^a)(y^a)$ 

Power property: 
$$\log_b m^p = p \log_b m$$
 Dividing Powers

Property of equality: If 
$$\log_b m = \log_b n$$
, 
$$\frac{\text{Dividing Two Powers of the Same Base:}}{(x^a)/(x^b) = x^{(a-b)}}$$
then  $m = n$  
$$\frac{\text{Dividing Powers of Different Bases:}}{(x/y)^a = (x^a)/(y^a)}$$

Change of base formula: 
$$\log_a n = \frac{\log_b n}{\log_b a}$$
 Slope-intercept form:  $y = mx + b$ 

Point-slope form: 
$$y = m(x - x_1) + y_1$$
  
Fractional exponent:  $\sqrt[b]{x^e} = x^{\frac{e}{b}}$ 

Standard form: 
$$Ax + By + C = 0$$
  
Negative Exponents:  $x^{-n} = 1/x^n$ 

## **Complex Fractions**

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

## **Example:**

$$\frac{-7 - \frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7 - \frac{6}{x+1}}{\frac{5}{x+1}} \cdot \frac{x+1}{x+1} = \frac{-7x - 7 - 6}{5} = \frac{-7x - 13}{5}$$

$$\frac{-2}{x} + \frac{3x}{x-4} = \frac{-2}{x} + \frac{3x}{x-4} = \frac{-2}{5 - \frac{1}{x-4}} \cdot \frac{3x}{x(x-4)} = \frac{-2(x-4) + 3x(x)}{5(x)(x-4) - 1(x)} = \frac{-2x + 8 + 3x^2}{5x^2 - 20x - x} = \frac{3x^2 - 2x + 8}{5x^2 - 21x}$$

Simplify each of the following.

$$1. \quad \frac{\frac{25}{a} - a}{5 + a}$$

$$2. \frac{2 - \frac{4}{x+2}}{5 + \frac{10}{x+2}}$$

3. 
$$\frac{4 - \frac{12}{2x - 3}}{5 + \frac{15}{2x - 3}}$$

4. 
$$\frac{\frac{x}{x+1} - \frac{1}{x}}{\frac{x}{x+1} + \frac{1}{x}}$$

$$5. \ \frac{1 - \frac{2x}{3x - 4}}{x + \frac{32}{3x - 4}}$$

#### **Function**

To evaluate a function for a given value, simply plug the value into the function for x.

**Recall:**  $(f \circ g)(x) = f(g(x)) OR f[g(x)]$  read "f of g of x" Means to plug the inside function (in this case g(x) ) in for x in the outside function (in this case, f(x)).

**Example:** Given  $f(x) = 2x^2 + 1$  and g(x) = x - 4 find f(g(x)).

$$f(g(x)) = f(x-4)$$

$$= 2(x-4)^{2} + 1$$

$$= 2(x^{2} - 8x + 16) + 1$$

$$= 2x^{2} - 16x + 32 + 1$$

$$f(g(x)) = 2x^{2} - 16x + 33$$

Let f(x) = 2x + 1 and  $g(x) = 2x^2 - 1$ . Find each.

6. 
$$f(2) =$$
\_\_\_\_\_

8. 
$$f(t+1) =$$
\_\_\_\_\_

9. 
$$f[g(-2)] =$$
\_\_\_\_\_

10. 
$$g[f(m+2)] =$$
\_\_\_\_\_\_

9. 
$$f[g(-2)] = \underline{\qquad}$$
 10.  $g[f(m+2)] = \underline{\qquad}$  11.  $\frac{f(x+h) - f(x)}{h} = \underline{\qquad}$ 

Let  $f(x) = \sin x$  Find each exactly.

12. 
$$f\left(\frac{\pi}{2}\right) =$$

13. 
$$f\left(\frac{2\pi}{3}\right) = \underline{\hspace{1cm}}$$

Let  $f(x) = x^2$ , g(x) = 2x + 5, and  $h(x) = x^2 - 1$ . Find each.

14. 
$$h[f(-2)] =$$
\_\_\_\_\_

14. 
$$h[f(-2)] =$$
 15.  $f[g(x-1)] =$  16.  $g[h(x^3)] =$ 

4

16. 
$$g[h(x^3)] =$$
\_\_\_\_\_

Find  $\frac{f(x+h)-f(x)}{h}$  for the given function f.

17. 
$$f(x) = 9x + 3$$

18. 
$$f(x) = 5 - 2x$$

**Intercepts and Points of Intersection** 

To find the x-intercepts, let y = 0 in your equation and solve. To find the y-intercepts, let x = 0 in your equation and solve.

**Example:**  $y = x^2 - 2x - 3$ 

$$\frac{x - \text{int. } (Let \ y = 0)}{0 = x^2 - 2x - 3}$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$
  
  $x = -1$  or  $x = 3$ 

$$x = -1 \text{ or } x = 3$$

x – intercepts (-1,0) and (3,0)

 $y - \text{int.} (Let \ x = 0)$   $y = 0^2 - 2(0) - 3$  y = -3 y - intercept (0, -3)

$$y = 0^2 - 2(0) - 3$$

$$y = -3$$

Find the x and y intercepts for each.

19. 
$$y = 2x - 5$$

20. 
$$y = x^2 + x - 2$$

21. 
$$y = x\sqrt{16 - x^2}$$

22. 
$$y^2 = x^3 - 4x$$

#### **Systems**

## Use substitution or elimination method to solve the system of equations.

## **Example:** $x^2 + y - 16x + 39 = 0$

$$x^{2} + y - 16x + 39$$
$$x^{2} - y^{2} - 9 = 0$$

## Elimination Method

$$2x^2 - 16x + 30 = 0$$

$$x^2 - 8x + 15 = 0$$

$$(x-3)(x-5)=0$$

$$x = 3$$
 and  $x = 5$ 

Plug x=3 and x=5 into one original

$$3^2 - y^2 - 9 = 0$$

$$^2 - y^2 - 9 = 0$$

$$-y^2 = 0$$

$$16 = y^2$$

$$y = 0$$

$$y = \pm 4$$

 $3^{2} - y^{2} - 9 = 0$   $5^{2} - y^{2} - 9 = 0$   $-y^{2} = 0$   $16 = y^{2}$  y = 0  $y = \pm 4$ Points of Intersection (5,4), (5,-4) and (3,0)

$$y^2 = -x^2 + 16x - 39$$

Substitution Method
Solve one equation for one variable.

$$y^{2} = -x^{2} + 16x - 39$$

$$x^{2} - (-x^{2} + 16x - 39) - 9 = 0$$
(1st equation solved for y)
$$x^{2} - (-x^{2} + 16x - 39) - 9 = 0$$
Plug what  $y^{2}$  is equal to into second equation.
$$2x^{2} - 16x + 30 = 0$$
(The rest is the same as previous example)
$$(x - 3)(x - 5) = 0$$

$$x = 3 \text{ or } x - 5$$

$$2x^2 - 16x + 30 - 0$$

$$x^2 - 8x + 15 = 0$$

$$(x-3)(x-5) = 0$$

$$x = 3 \text{ or } x - 5$$

Find the point(s) of intersection of the graphs for the given equations.

$$23. \qquad \begin{aligned} x + y &= 8 \\ 4x - y &= 7 \end{aligned}$$

$$24. \qquad x^2 + y = 6$$
$$x + y = 4$$

25. 
$$x^{2} - 4y^{2} - 20x - 64y - 172 = 0$$
$$16x^{2} + 4y^{2} - 320x + 64y + 1600 = 0$$

## **Interval Notation**

26. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \le 4$		
	[-1,7)	
		<b>←</b>

Solve each equation. State your answer in BOTH interval notation and graphically.

27. 
$$2x-1 \ge 0$$

28. 
$$-4 \le 2x - 3 < 4$$

29. 
$$\frac{x}{2} - \frac{x}{3} > 5$$

## **Domain and Range**

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. 
$$f(x) = x^2 - 5$$

31. 
$$f(x) = -\sqrt{x+3}$$

$$32. \quad f(x) = 3\sin x$$

30. 
$$f(x) = x^2 - 5$$
 31.  $f(x) = -\sqrt{x+3}$  32.  $f(x) = 3\sin x$  33.  $f(x) = \frac{2}{x-1}$ 

## **Inverses**

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

**Example:** 

$$f(x) = \sqrt[3]{x+1}$$
 Rewrite  $f(x)$  as y  
 $y = \sqrt[3]{x+1}$  Switch x and y  
 $x = \sqrt[3]{y+1}$  Solve for your new y  
 $(x)^3 = (\sqrt[3]{y+1})^3$  Cube both sides  
 $x^3 = y+1$  Simplify  
 $y = x^3 - 1$  Solve for y

 $f^{-1}(x) = x^3 - 1$  Rewrite in inverse notation

Find the inverse for each function.

**34.** 
$$f(x) = 2x + 1$$

**35.** 
$$f(x) = \frac{x^2}{3}$$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: f(g(x)) = g(f(x)) = x

#### **Example:**

If:  $f(x) = \frac{x-9}{4}$  and g(x) = 4x + 9 show f(x) and g(x) are inverses of each other.

$$g(f(x)) = 4\left(\frac{x-9}{4}\right) + 9$$

$$= x - 9 + 9$$

$$= x$$

$$= x$$

$$f(g(x)) = \frac{(4x+9)-9}{4}$$

$$= \frac{4x+9-9}{4}$$

$$= \frac{4x}{4}$$

$$= x$$

f(g(x)) = g(f(x)) = x therefore they are inverses of each other.

Prove f and g are inverses of each other.

**36.** 
$$f(x) = \frac{x^3}{2}$$
  $g(x) = \sqrt[3]{2x}$ 

**37.** 
$$f(x) = 9 - x^2, x \ge 0$$
  $g(x) = \sqrt{9 - x}$ 

## **Equation of a line**

- **Slope intercept form:** y = mx + b **Vertical line:** x = c (slope is undefined)
- **Point-slope form:**  $y y_1 = m(x x_1)$  **Horizontal line:** y = c (slope is 0)
- 38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.
- 39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.
- 40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.
- 41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.
- 42. Find the equation of a line passing through the point (2, 8) and parallel to the line  $y = \frac{5}{6}x 1$ .

- 43. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).
- 44. Find the equation of a line passing through the points (-3, 6) and (1, 2).
- 45. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

## **Radian and Degree Measure**

Use  $\frac{180^{\circ}}{\pi \, radians}$  to get rid of radians and convert to degrees.

Use  $\frac{\pi \, radians}{180^{\circ}}$  to get rid of degrees and convert to radians.

- 46. Convert to degrees:
- a.  $\frac{5\pi}{6}$
- b.  $\frac{4\pi}{5}$

c. 2.63 radians

- 47. Convert to radians:
- a. 45°
- b. -17°

c. 237°

## **Angles in Standard Position**

48. Sketch the angle in standard position.

a. 
$$\frac{11\pi}{6}$$

b. 230°

c.  $-\frac{5\pi}{3}$ 

d. 1.8 radians

## **Reference Triangles**

- 49. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.
- a.  $\frac{2}{3}\pi$

b. 225°

c.  $-\frac{\pi}{4}$ 

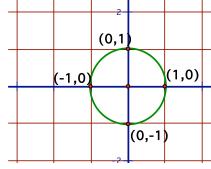
d.  $30^{\circ}$ 

## **Unit Circle**

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

**Example:**  $\sin 90^{\circ} = 1$ 

 $\cos\frac{\pi}{2} = 0$ 



- 50.
- a.)  $\sin 180^{\circ}$

b.) cos 270°

- c.)  $\sin(-90^{\circ})$
- d.)  $\sin \pi$

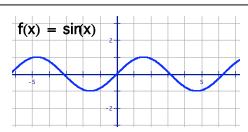
(-1,0) (1,0) (0,-1)

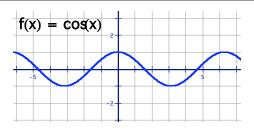
(0,1]

e.)  $\cos 360^{\circ}$ 

f.)  $\cos(-\pi)$ 

**Graphing Trig Functions** 





 $y = \sin x$  and  $y = \cos x$  have a period of  $2\pi$  and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For  $f(x) = A\sin(Bx + C) + K$ , A = amplitude,  $\frac{2\pi}{B} = \text{period}$ ,

 $\frac{C}{B}$  = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Graph two complete periods of the function.

51. 
$$f(x) = 5 \sin x$$

52. 
$$f(x) = \sin 2x$$

$$53. \ f(x) = -\cos\left(x - \frac{\pi}{4}\right)$$

$$54. \ f(x) = \cos x - 3$$

**Trigonometric Equations:** 

Solve each of the equations for  $0 \le x < 2\pi$ . Isolate the variable, sketch a reference triangle, find all the solutions within the given domain,  $0 \le x < 2\pi$ . Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the beginning of the packet.)

55. 
$$\sin x = -\frac{1}{2}$$

$$56. \ \ 2\cos x = \sqrt{3}$$

$$57. \cos 2x = \frac{1}{\sqrt{2}}$$

58. 
$$\sin^2 x = \frac{1}{2}$$

59. 
$$\sin 2x = -\frac{\sqrt{3}}{2}$$

$$60. \ \ 2\cos^2 x - 1 - \cos x = 0$$

61. 
$$4\cos^2 x - 3 = 0$$

62. 
$$\sin^2 x + \cos 2x - \cos x = 0$$

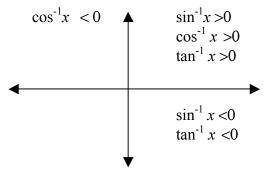
## **Inverse Trigonometric Functions:**

**Recall:** Inverse Trig Functions can be written in one of ways:

$$\arcsin(x)$$

$$\sin^{-1}(x)$$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.

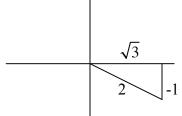


**Example:** 

Express the value of "y" in radians.

$$y = \arctan \frac{-1}{\sqrt{3}}$$

Draw a reference triangle.



This means the reference angle is 30° or  $\frac{\pi}{6}$ . So,  $y = -\frac{\pi}{6}$  so that it falls in the interval from

14

$$\frac{-\pi}{2} < y < \frac{\pi}{2}$$

$$\frac{-\pi}{2} < y < \frac{\pi}{2}$$
 Answer:  $y = -\frac{\pi}{6}$ 

For each of the following, express the value for "y" in radians.

76. 
$$y = \arcsin \frac{-\sqrt{3}}{2}$$

77. 
$$y = \arccos(-1)$$

78. 
$$y = \arctan(-1)$$

**Example:** Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.

$$\frac{\sqrt{61}}{\theta}$$

$$\cos\theta = \frac{6}{\sqrt{61}}$$

For each of the following give the value without a calculator.

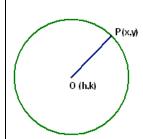
63. 
$$\tan\left(\arccos\frac{2}{3}\right)$$

64. 
$$\sec\left(\sin^{-1}\frac{12}{13}\right)$$

65. 
$$\sin\left(\arctan\frac{12}{5}\right)$$

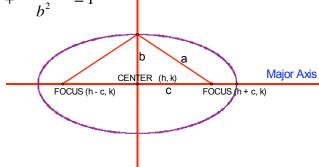
66. 
$$\sin\left(\sin^{-1}\frac{7}{8}\right)$$

**Circles and Ellipses** 



$$r^2 = (x-h)^2 + (y-k)^2$$

$$r^{2} = (x-h)^{2} + (y-k)^{2} \qquad \frac{(x-h)^{2}}{a^{2}} + \frac{(y-k)^{2}}{b^{2}} = 1$$



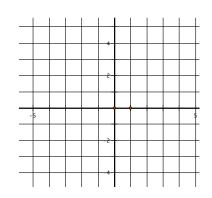
For a circle centered at the origin, the equation is  $x^2 + y^2 = r^2$ , where **r** is the radius of the circle.

For an ellipse centered at the origin, the equation is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , where **a** is the distance from the center to the

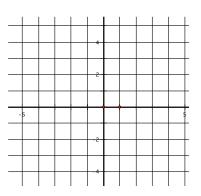
ellipse along the x-axis and **b** is the distance from the center to the ellipse along the y-axis. If the larger number is under the  $y^2$  term, the ellipse is elongated along the y-axis. For our purposes in Calculus, you will not need to locate the foci.

Graph the circles and ellipses below:

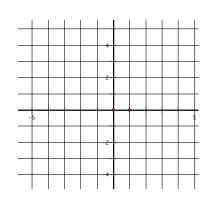
$$67. \ x^2 + y^2 = 16$$



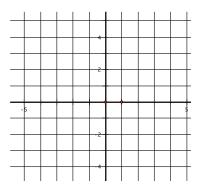
68. 
$$x^2 + y^2 = 5$$



$$69. \ \frac{x^2}{1} + \frac{y^2}{9} = 1$$



70. 
$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$



## **Vertical Asymptotes**

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

71. 
$$f(x) = \frac{1}{x^2}$$

72. 
$$f(x) = \frac{x^2}{x^2 - 4}$$

73. 
$$f(x) = \frac{2+x}{x^2(1-x)}$$

## **Horizontal Asymptotes**

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

**Case II.** Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

**Case III**. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

**Determine all Horizontal Asymptotes.** 

74. 
$$f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$$

75. 
$$f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$$

76. 
$$f(x) = \frac{4x^5}{x^2 - 7}$$

## **Laws of Exponents**

Write each of the following expressions in the form  $ca^pb^q$  where c, p and q are constants (numbers).

$$75. \qquad \frac{(2a^2)^3}{b}$$

76. 
$$\sqrt{9ab^3}$$

77. 
$$\frac{a(\frac{2}{b})}{\frac{3}{a}}$$

(Hint: 
$$\sqrt{x} = x^{1/2}$$
)

$$78. \qquad \frac{ab-a}{b^2-b}$$

79. 
$$\frac{a^{-1}}{(b^{-1})\sqrt{a}}$$

$$80. \left(\frac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\right)^{2} \left(\frac{b^{\frac{3}{2}}}{a^{\frac{1}{2}}}\right)$$

## **Laws of Logarithms**

Simplify each of the following:

81. 
$$\log_2 5 + \log_2(x^2 - 1) - \log_2(x - 1)$$
 82.  $2\log_2 9 - \log_2 3$ 

82. 
$$2\log_2 9 - \log_2 3$$

83. 
$$3^{2\log_3 5}$$

84. 
$$\log_{10}(10^{\frac{1}{2}})$$

85. 
$$\log_{10}(\frac{1}{10^x})$$

$$86. \ 2\log_{10}\sqrt{x} + \log_{10}x^{\frac{1}{3}}$$

## **Solving Exponential and Logarithmic Equations**

Solve for x. (DO NOT USE A CALCULATOR)

87. 
$$5^{(x+1)} = 25$$

$$88. \quad \frac{1}{3} = 3^{2x+2}$$

89. 
$$\log_2 x^2 = 3$$

87. 
$$5^{(x+1)} = 25$$
 88.  $\frac{1}{3} = 3^{2x+2}$  89.  $\log_2 x^2 == 3$  90.  $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$ 

## **Factor Completely**

91. 
$$x^6 - 16x^6$$

91. 
$$x^6 - 16x^4$$
 92.  $4x^3 - 8x^2 - 25x + 50$  93.  $8x^3 + 27$  94.  $x^4 - 1$ 

93. 
$$8x^3 + 27$$

94. 
$$x^4 - 1$$

## Solve the following equations for the indicated variables:

95. 
$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$
, for a.

95. 
$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$
, for a. 96.  $V = 2(ab + bc + ca)$ , for a. 97.  $A = 2\pi r^2 + 2\pi rh$ , for positive r.

97. 
$$A = 2\pi r^2 + 2\pi rh$$
, for positive r.

Hint: use quadratic formula

98. 
$$A = P + xrP$$
, for P

$$99. 2x - 2yd = y + xd, for d$$

98. 
$$A = P + xrP$$
, for  $P$  99.  $2x - 2yd = y + xd$ , for  $d$  100.  $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$ , for  $x$ 

## Solve the equations for x:

101. 
$$4x^2 + 12x + 3 = 0$$

101. 
$$4x^2 + 12x + 3 = 0$$
 102.  $2x + 1 = \frac{5}{x + 2}$ 

103. 
$$\frac{x+1}{x} - \frac{x}{x+1} = 0$$

## **Polynomial Division**

104. 
$$(x^5 - 4x^4 + x^3 - 7x + 1) \div (x + 2)$$

105. 
$$(x^5 - x^4 + x^3 + 2x^2 - x + 4) \div (x^3 + 1)$$

## **AP CALCULUS**

## **SUMMER WORK PART 2**

Graph the parent function of each set using your calculator. Draw a quick sketch on your paper of each additional equation in the family. Check your sketch with the graphing calculator. You may have to look up how to use a graphing calculator to complete this section. You also may need to download a digital TI-84 calculator to complete this assignment if you do not have one.

1) Parent Function: 
$$y = x^2$$

a) 
$$y = x^2 - 5$$

b) 
$$y = x^2 + 3$$

c) 
$$y = (x-10)^2$$

d) 
$$y = (x+8)^2$$

e) 
$$y = 4x^2$$

f) 
$$y = 0.25x^2$$

g) 
$$y = -x^2$$

h) 
$$y = -(x+3)^2 + 6$$

I) 
$$y = (x+4)^2 - 8$$

j) 
$$y = -2(x+1)^2 + 4$$

k) 
$$y = \frac{1}{3}(x-6)^2 - 6$$

$$y = -3(x+2)^2 - 2$$

2) Parent Function: 
$$y = sin(x)$$
 (set mode to RADIANS)

a) 
$$y = \sin(2x)$$

b) 
$$y = \sin(x) - 2$$

c) 
$$y = 2 \sin(x)$$

d) 
$$y = 2\sin(2x) + 2$$

3) Parent Function: 
$$y = cos(x)$$

a) 
$$y = cos(3x)$$

b) 
$$y = cos(x/2)$$

c) 
$$y = 2\cos(x) + 2$$

d) 
$$y = -2\cos(x) - 1$$

4) Parent Function: 
$$y = x^3$$

a) 
$$y = x^3 + 2$$

b) 
$$y = -x^3$$

b) 
$$y = x^3 - 5$$

c) 
$$y = -x^3 + 3$$

e) 
$$y = (x-4)^3$$

f) 
$$y = (x-1)^3 - 4$$

g) 
$$y = -2(x+2)^3 + 1$$

h) 
$$y = x^3 + x$$

5) Parent Function: 
$$y = \sqrt{x}$$

a) 
$$y = \sqrt{x} - 2$$

b) 
$$y = \sqrt{-x}$$

c) 
$$y = \sqrt{x} + 5$$

$$y = \sqrt{6 - x}$$

e) 
$$y = -\sqrt{x}$$

$$f) y = -\sqrt{-x}$$

g) 
$$y = \sqrt{x+2}$$

$$h) y = \sqrt{2x - 6}$$

i) 
$$y = -2\sqrt{x}$$

$$j) y = -\sqrt{4 - x}$$

6) Parent Function: 
$$y = In(x)$$

a) 
$$y = ln(x+3)$$

b) 
$$y = ln(x) + 3$$

c) 
$$y = In(x-2)$$

d) 
$$y = ln(-x)$$

e) 
$$y = -ln(x)$$

f) 
$$y = ln(|x|)$$

g) 
$$y = \ln(2x) - 4$$

h) 
$$y = -3ln(x) + 1$$

7) Parent Function: 
$$y = e^x$$

a) 
$$y = e^{2x}$$

b) 
$$y = e^{x-2}$$

c) 
$$y = e^{2-x}$$

d) 
$$y = e^{2x} + 3$$

e) 
$$y = -e^x$$

f) 
$$y = e^{-x}$$

g) 
$$y = 2 - e^x$$

h) 
$$y = e^{0.5x}$$

8) Parent Function 
$$y = a^x$$

a) 
$$y = 5^x$$

b) 
$$y = 2^{x}$$

c) 
$$y = 3^{-x}$$

d) 
$$y = \frac{1}{2}^{x}$$

e) 
$$y = 4^{x-3}$$

f) 
$$y = 2^{x-3} + 2$$

9) Parent Function: 
$$y = 1/x$$

a) 
$$y = 1/(x-2)$$

b) 
$$y = -1/x$$

c) 
$$y = 1/(x+4)$$

d) 
$$y = 2/(5-x)$$

10) Parent Function: 
$$y = [x]$$

Note: [x] is the IntegerPart of x. On the TI-83/84 it is found in the MATH menu, NUM submenu.

a) 
$$y = [x] + 2$$

b) 
$$y = [x-3]$$

c) 
$$y = [3x]$$

d) 
$$y = [0.25x]$$

e) 
$$y = 3 - [x]$$

e) 
$$y = 2[x] - 1$$

11) Resize your viewing window to  $[0,1] \times [0,1]$ . Graph all of the following functions in the same window. List the functions from the highest graph to the lowest graph. How do they compare for values of x > 1?

a) 
$$y = x^2$$

b) 
$$y = x^{3}$$

c) 
$$y = \sqrt{x}$$

d) 
$$y = x^{2/3}$$

e) 
$$y = |x|$$

f) 
$$y = x^4$$

12) Given: 
$$f(x) = x^4-3x^3+2x^2-7x-11$$
  
Find all roots to the nearest 0.001

13) Given:  $f(x) = 3 \sin 2x - 4x + 1$  from  $[-2\pi, 2\pi]$  Find all roots to the nearest 0.001.

Note: All trig functions are done in radian mode.

- 14) Given:  $f(x) = 0.7x^2 + 3.2x + 1.5$  Find all roots to the nearest 0.001.
- 15) Given:  $f(x) = x^4 8x^2 + 5$ Find all roots to the nearest 0.001.
- 16) Given:  $f(x) = x^3 + 3x^2 10x 1$ Find all roots to the nearest 0.001
- 17) Given:  $f(x) = 100x^3 203x^2 + 103x 1$ Find all roots to the nearest 0.001
- 18) Given: f(x) = |x-3| + |x| 6Find all roots to the nearest 0.001
- 19) Given: f(x) = |x| |x-6| = 0Find all roots to the nearest 0.001

Solve the following inequalities

- 20)  $x^2 x 6 > 0$
- 21)  $x^2 2x 5 \ge 3$
- 22)  $x^3 4x < 0$

For each of the following (problems 23-26)

- a) Sketch the graph of f(x)
- b) Sketch the graph of |f(x)|
- c) Sketch the graph of f(|x|)
- d) Sketch the graph of f(2x)
- e) Sketch the graph of 2f(x)
- 23) f(x) = 2x+3
- 24)  $f(x) = x^2 5x 3$
- 25)  $f(x) = 2\sin(3x)$
- 26)  $f(x) = -x^3 2x^2 + 3x 4$
- 27) Let  $f(x) = \sin x$ Let  $g(x) = \cos x$ 
  - a) Sketch the graph of f<sup>2</sup>
  - b) Sketch the graph of g<sup>2</sup>
  - c) Sketch the graph of  $f^2 + g^2$

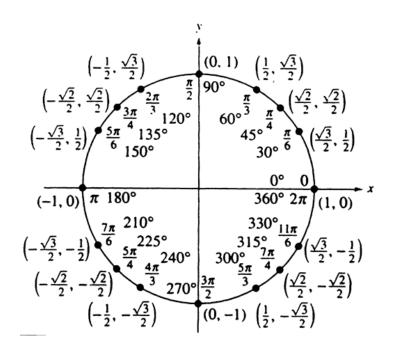
28) Given: 
$$f(x) = 3x+2$$
  
  $g(x) = -4x-2$ 

29) Given: 
$$f(x) = x^2 - 5x + 2$$
  
  $g(x) = 3-2x$ 

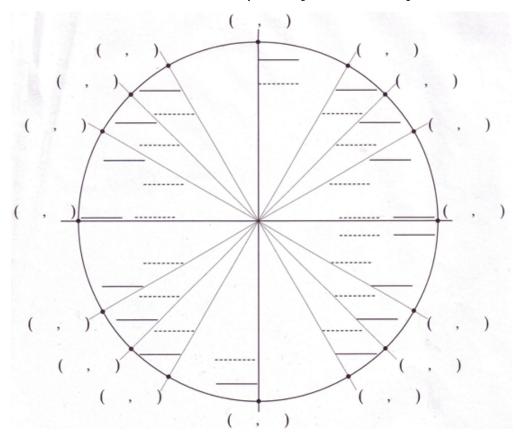
Find the coordinates of any points of intersection.

- 30) How many times does the graph of y = 0.1x intersect the graph of  $y = \sin(2x)$ ?
- 31) Given:  $f(x) = x^4 7x^3 + 6x^2 + 8x + 9$ 
  - a) Determine the x- and y-coordinates of the lowest point on the graph.
  - b) Size the x-window from [-10,10]. Find the highest and lowest values of f(x) over the interval  $-10 \le x \le 10$

#### YOU MUST HAVE THE UNIT CIRCLE MEMORIZED



Here is a blank Unit Circle that you can print and use to practice



You must also memorize this chart. You will have a quiz on day 2 of class.

Degrees	Radians	sin θ	cos θ	tan θ	cot θ	sec θ	$\csc \theta$
0°	0	0	1	0	Undefined	1	Undefined
30°	π/6	1/2	√3 <i>1</i> 2	√3/3	√3	2√3/3	2
45°	π/4	√212	√212	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	π/3	√3 <i>1</i> 2	1/2	√3	√3 /3	2	2√3/3
90°	π/2	1	0	Undefined	0	Undefined	1
120°	2π/3	√3/2	-1/2	-√3	-√3 <i>r</i> 3	-2	2√3/3
135°	$3\pi/4$	√212	-√2 <i>1</i> 2	-1	-1	-√2	$\sqrt{2}$
150°	5π/6	1/2	-√3 <i>1</i> 2	-√3 <i>r</i> 3	-√3	-2√3/3	2
180°	π	0	-1	0	Undefined	-1	Undefined
210°	7π/6	-1/2	-√3/2	√3 /3	√3	-2√3 <i>1</i> 3	-2
225°	5π/4	-√212	-√212	1	1	-√2	-√2
240°	4π/3	-√3 <i>r</i> 2	-1/2	√3	√3 <i>1</i> 3	-2	-2√3/3
270°	3π/2	-1	0	Undefined	0	Undefined	-1
300°	5π/3	-√3/2	1/2	-√3	-√3 <i>1</i> 3	2	-2√3 /3
315°	7π/4	-√212	V212	-1	-1	$\sqrt{2}$	-√2
330°	11π/6	-1/2	√312	-√3/3	-√3	2√3/3	-2
360°	2π	0	1	0	Undefined	1	Undefined

Name	Ratio	Notation
Sine	opposite/hypotenuse	Sin(θ)
Cosine	adjacent/hypotenuse	Cos(θ)
Tangent	opposite/adjacent	Tan(θ)
Cosecant (1/Sine)	hypotenuse/opposite	Cosec(θ) or csc(θ)
Secant (1/Cosine)	hypotenuse/adjacent	Sec(θ)
Cotangent (1/Tangent)	adjacent/opposite	Cot(θ)